6-1 The Polygon Angle-Sum Theorems

Do Now

Lesson Presentation

Exit Ticket

6-1 The Polygon Angle-Sum Theorems

Do Now #3

1. A ___?___ is a three-sided polygon.

2. A ___?___ is a four-sided polygon.

Evaluate each expression for *n* = 6.

3. $(n - 4)\ 12$

4. $(n - 3)\ 90$

Solve for *a*.

5. $12a + 4a + 9a = 100$

6-1 The Polygon Angle-Sum Theorems

Do Now #3

1. A ___?___ is a three-sided polygon. triangle

2. A ___?___ is a four-sided polygon. quadrilateral

Evaluate each expression for $n = 6$.

3. $(n - 4)\ 12$ 24

4. $(n - 3)\ 90$ 270

Solve for a.

5. $12a + 4a + 9a = 100$ $a = 4$

6-1 The Polygon Angle-Sum Theorems

Connect to Mathematical Ideas (1)(F)

By the end of today's lesson,

SWBAT

- Classify polygons based on their sides and angles.
- Find and use the measures of interior and exterior angles of polygons.

6-1 The Polygon Angle-Sum Theorems

Vocabulary

equiangular polygon

equilateral polygon

regular polygon

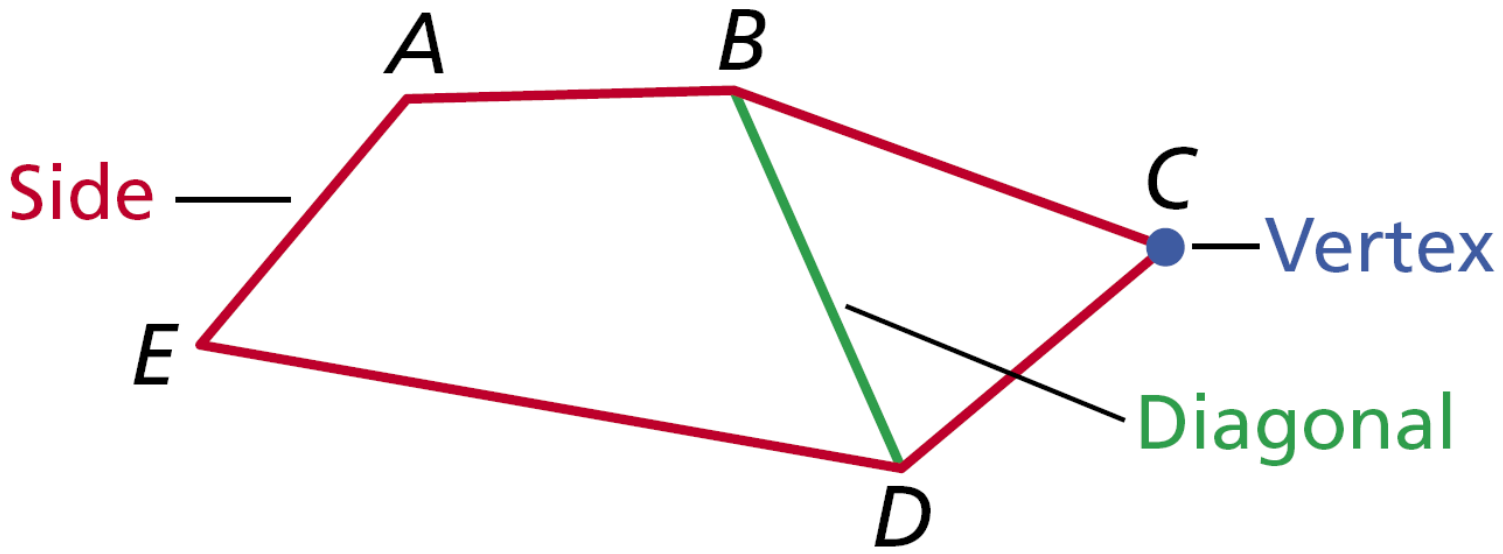
number sense

6-1 The Polygon Angle-Sum Theorems

In Unit 1, you learned the definition of a polygon. Now you will learn about the parts of a polygon and about ways to classify polygons.

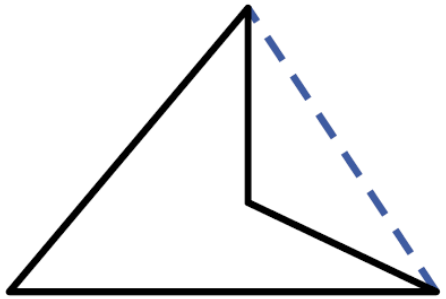
6-1 The Polygon Angle-Sum Theorems

Each segment that forms a polygon is a **side of the polygon**. The common endpoint of two sides is a **vertex of the polygon**. A segment that connects any two nonconsecutive vertices is a **diagonal**.

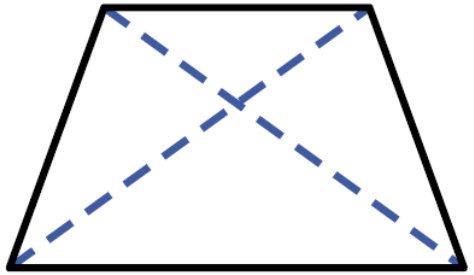


6-1 The Polygon Angle-Sum Theorems

A polygon is **concave** if any part of a diagonal contains points in the exterior of the polygon. If no diagonal contains points in the exterior, then the polygon is **convex**. A regular polygon is always convex.



Concave quadrilateral



Convex quadrilateral

6-1 The Polygon Angle-Sum Theorems

You can name a polygon by the number of its sides. The table shows the names of some common polygons.

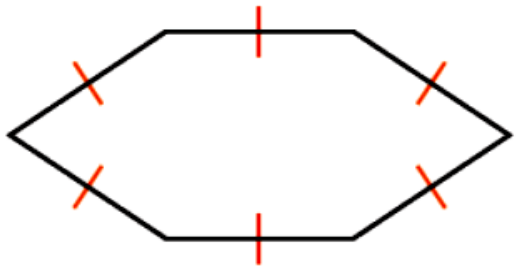
Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	*n*-gon

6-1 The Polygon Angle-Sum Theorems

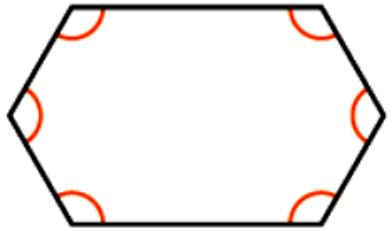
take note

Key Concept Classifying Polygons Based on Sides and Angles

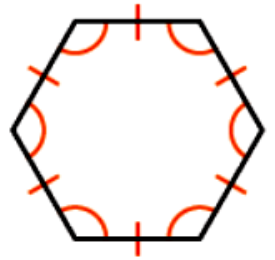
An **equilateral polygon** is a polygon with all sides congruent.



An **equiangular polygon** is a polygon with all angles congruent.



A **regular polygon** is a polygon that is both equilateral and equiangular.



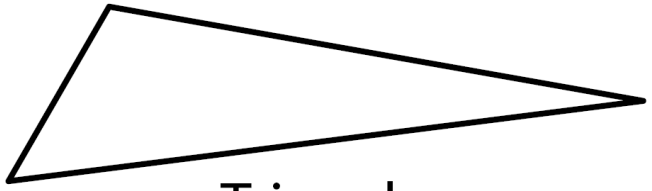
6-1 The Polygon Angle-Sum Theorems

Remember!

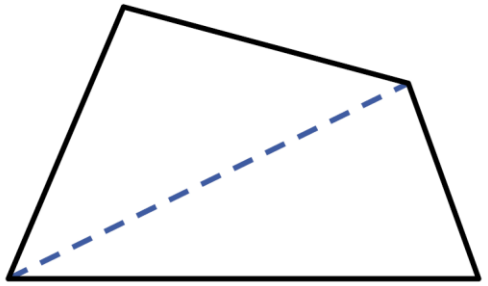
A polygon is a closed plane figure formed by three or more segments that intersect only at their endpoints.

6-1 The Polygon Angle-Sum Theorems

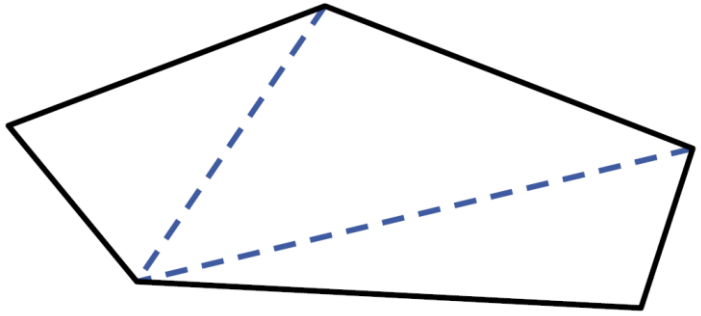
To find the sum of the interior angle measures of a convex polygon, draw all possible diagonals from one vertex of the polygon. This creates a set of triangles. The sum of the angle measures of all the triangles equals the sum of the angle measures of the polygon.



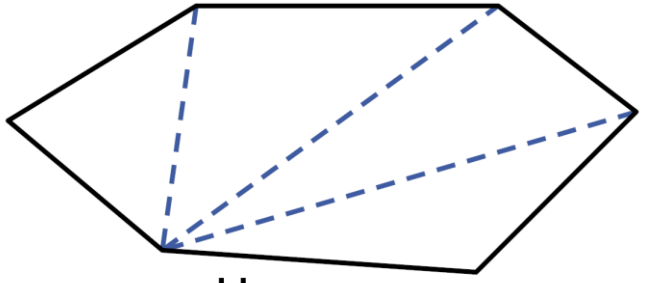
Triangle



Quadrilateral



Pentagon

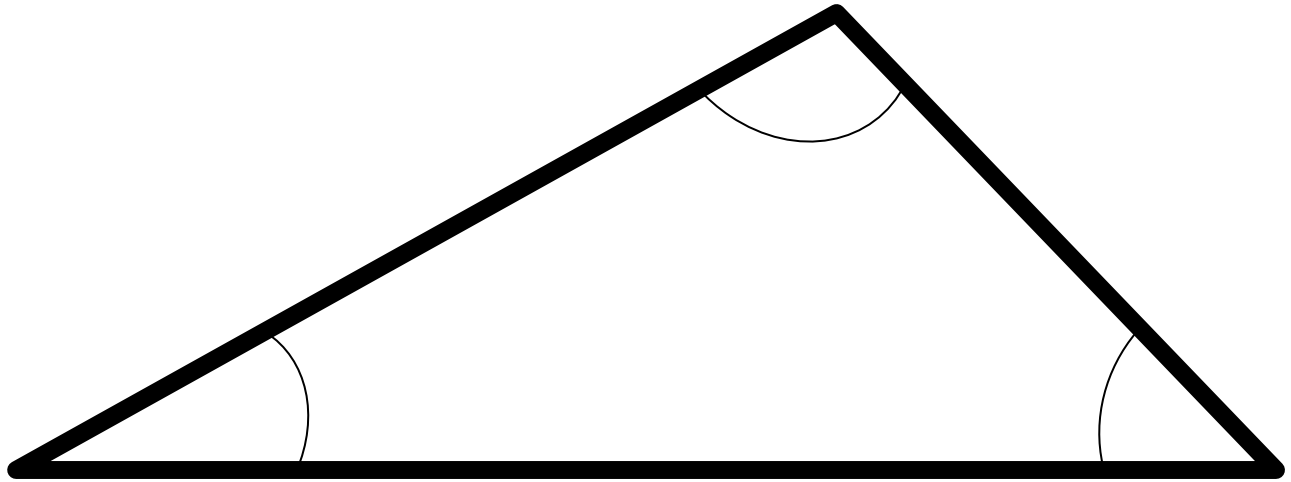


Hexagon

6-1 The Polygon Angle-Sum Theorems

Remember!

By the Triangle Sum Theorem, the sum of the interior angle measures of a triangle is 180°.



6-1 The Polygon Angle-Sum Theorems

Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	$(1)180° = 180°$
Quadrilateral	4	2	$(2)180° = 360°$
Pentagon	5	3	$(3)180° = 540°$
Hexagon	6	4	$(4)180° = 720°$
n-gon	n	$n - 2$	$(n - 2)180°$

6-1 The Polygon Angle-Sum Theorems

In each convex polygon, the number of triangles formed is two less than the number of sides n. So the sum of the angle measures of all these triangles is $(n - 2)180°$.

take note

Theorem 6-1 Polygon Angle-Sum Theorem

The sum of the measures of the interior angles of an n-gon is $(n - 2)180$.

For a proof of Theorem 6-1, see the Reference section on page 683.

Corollary to the Polygon Angle-Sum Theorem

The measure of each interior angle of a regular n-gon is $\frac{(n - 2)180}{n}$.

You will prove the Corollary to the Polygon Angle-Sum Theorem in Exercise 16.

6-1 The Polygon Angle-Sum Theorems

Example 1: Finding a Polygon Angle Sum

What is the sum of the interior angle measures of a heptagon?

$S_n = (n - 2)180^\circ$ *Polygon Angle-Sum Theorem*

$= (7 - 2)180^\circ$ *Substitute 7 for n*

$= (5)180^\circ$ *Simplify*

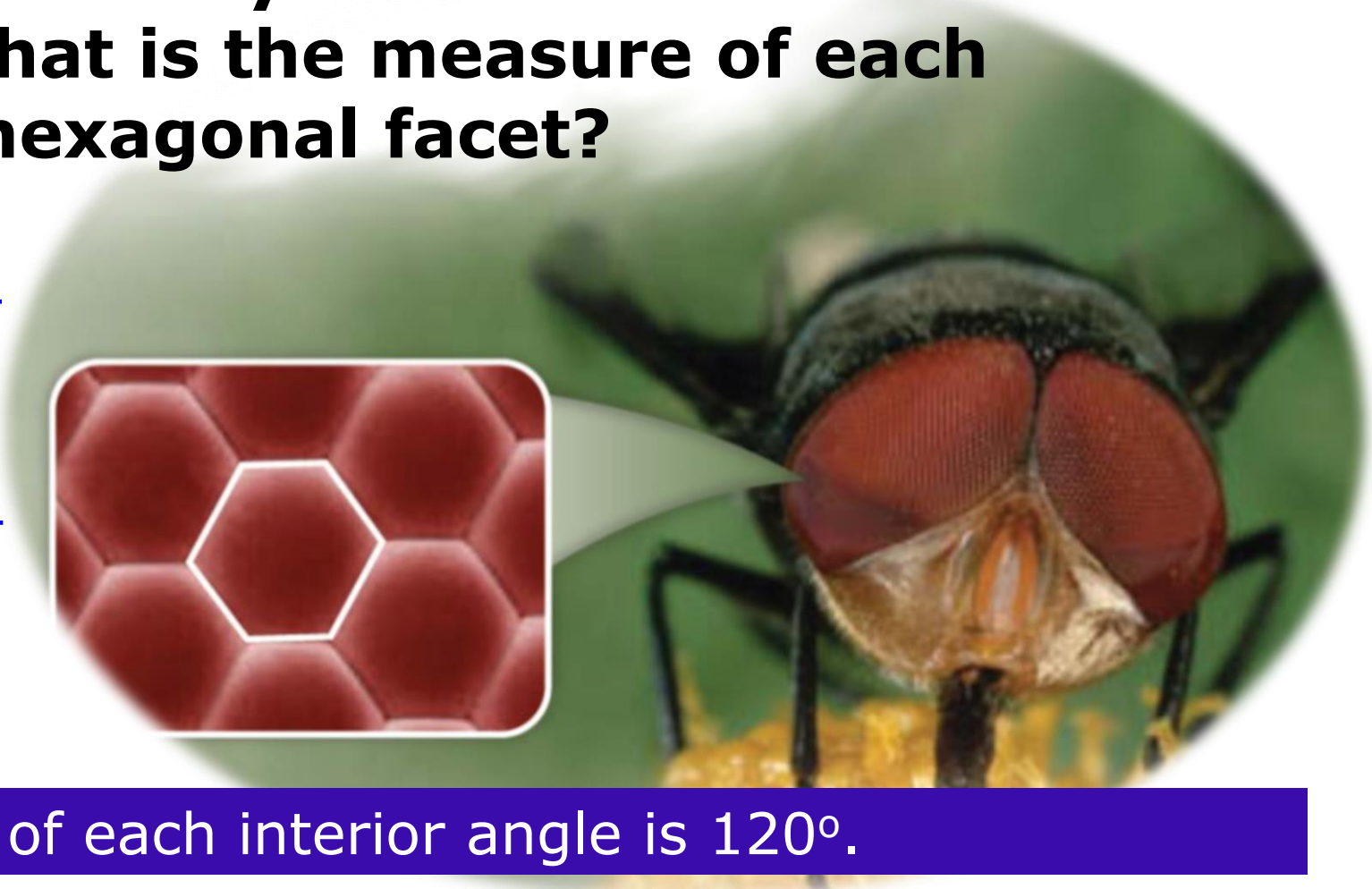
$S_n = 900^\circ$

The sum of the interior angle measures of the heptagon is 900°.

6-1 The Polygon Angle-Sum Theorems

Example 2: Using the Polygon Angle-Sum

Biology **The common housefly, *Musca domestica*, has eyes that consist of approximately 4000 facets. Each facet is a regular hexagon. What is the measure of each interior angle in one hexagonal facet?**



$$m(\text{each } \angle) = \frac{(n-2)180^{\circ}}{n}$$

$$= \frac{(6-2)180^{\circ}}{6}$$

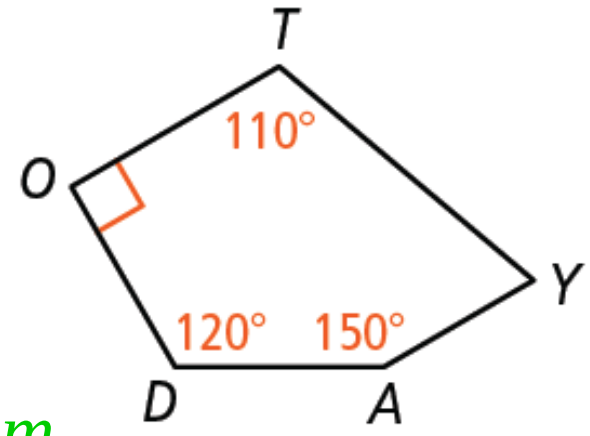
$$= \frac{(4)180^{\circ}}{6}$$

The measure of each interior angle is 120°.

6-1 The Polygon Angle-Sum Theorems

Example 3: Using the Polygon Angle-Sum

What is $m\angle Y$ in pentagon *TODAY* ?



How many side is in the given polygon? $n = 5$

$S_n = (n - 2)180°$ *Polygon Angle-Sum Theorem*

$m\angle T + m\angle O + m\angle D + m\angle A + m\angle Y = (5 - 2)180$ *Substitute 5 for n*

$110 + 90 + 120 + 150 + m\angle Y = (3)180$ *Substitute*

$470 + m\angle Y = 540$ *Simplify*

$m\angle Y = 70$ *Subtract 470 from each side*

Example 4: Investigating the Exterior Angles of Polygons

A **Which of the following tools would be appropriate to use for investigating the exterior angles of polygons? Explain your choice.**

- **Ruler**
- **Protractor**
- **Graphing Calculator**

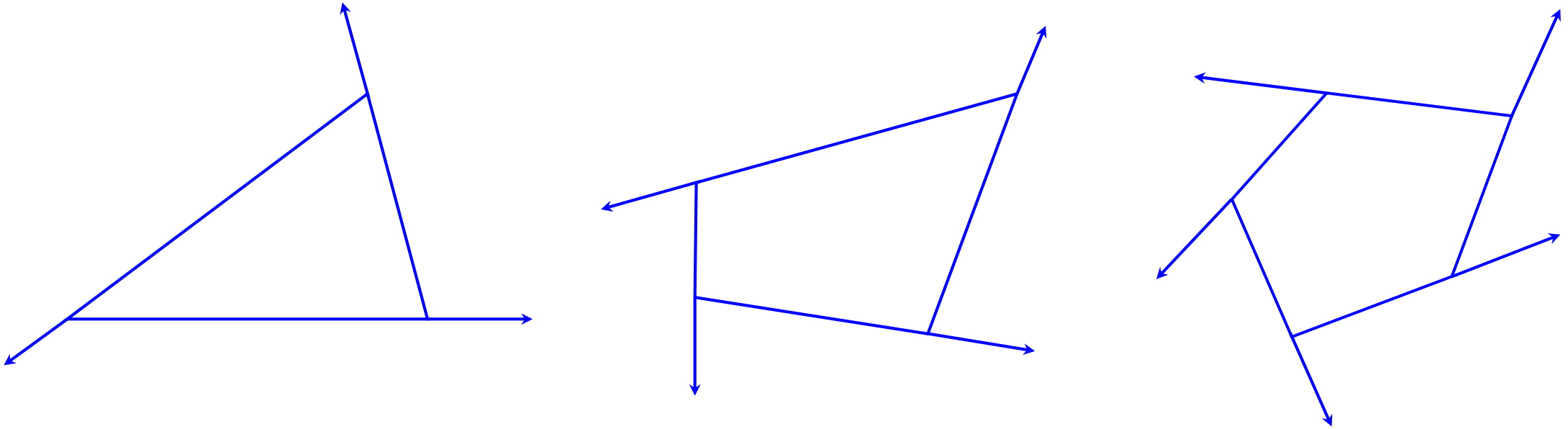
because protractors are use to measure angles

6-1 The Polygon Angle-Sum Theorems

Example 4: Investigating the Exterior Angles of Polygons

B **Draw an exterior angle at each vertex of three different polygons. Investigate patterns and write a conjecture about the exterior angles.**

Step 1: Draw three different polygons. Then draw the exterior angles at each vertex of the polygons as shown.

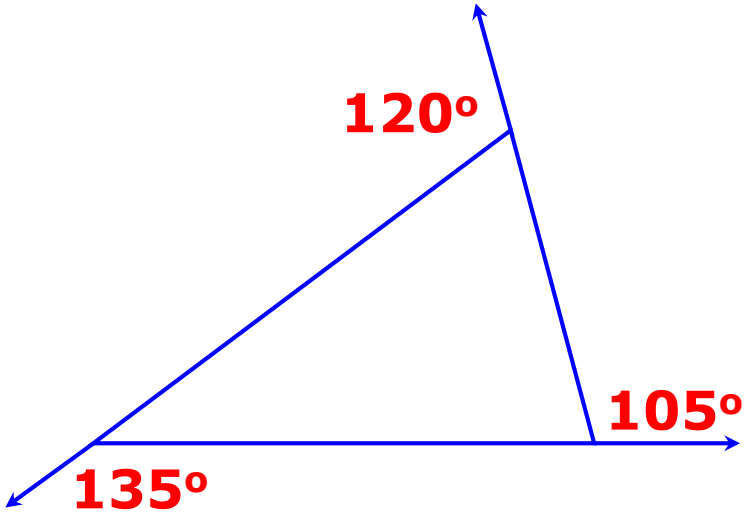


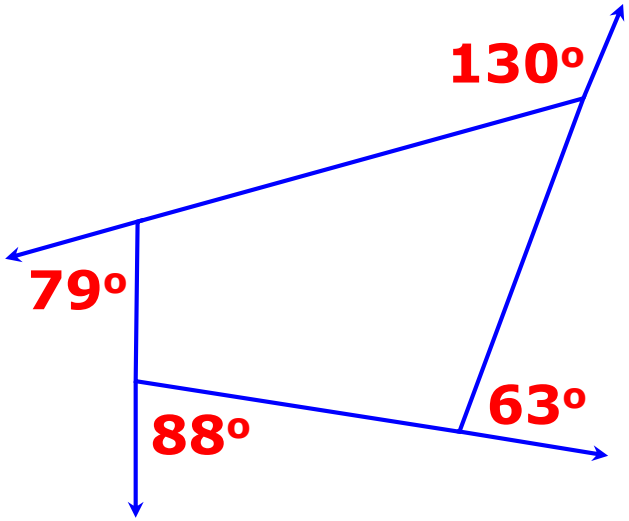
6-1 The Polygon Angle-Sum Theorems

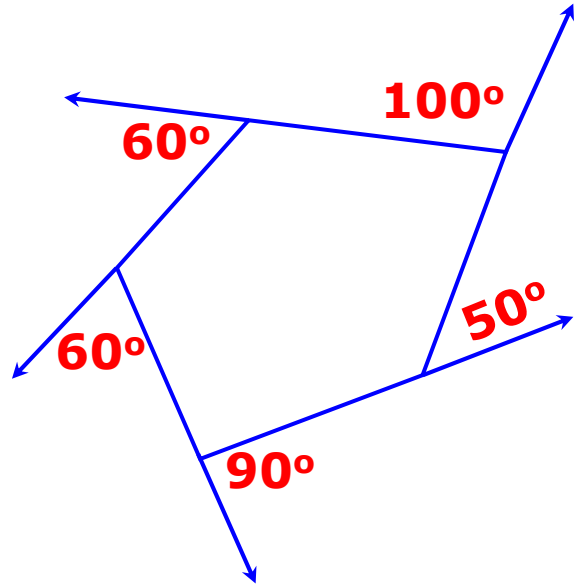
Example 4: Investigating the Exterior Angles of Polygons

B **Step 2: Use the protractor to measure the exterior angles of each polygon. Observe any patterns. Write a conjecture about the exterior angles of polygons.**

Conjecture: The sum of measures of the exterior angles of a polygon, one at each vertex, is 360.







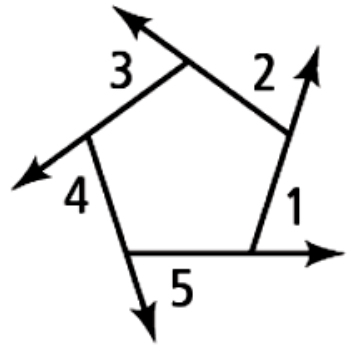
6-1 The Polygon Angle-Sum Theorems

take note

Theorem 6-2 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$.



You will prove Theorem 6-2 in Exercise 9.

6-1 The Polygon Angle-Sum Theorems

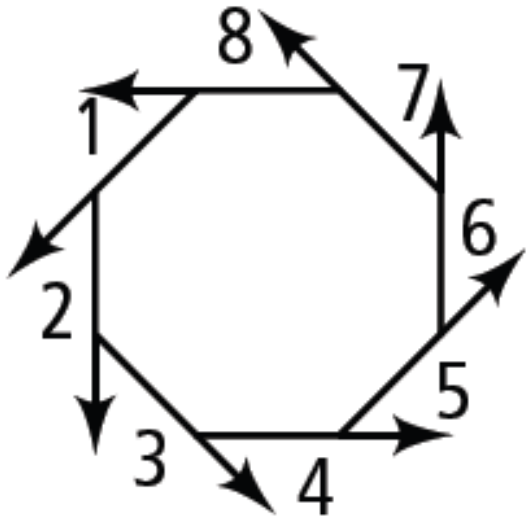
Example 5: Finding an Exterior Angle Measure

What is $m\angle 1$ in the regular octagon at the right ?

360° *Polygon Exterior Angle-Sum Theorem*

$m\angle 1 = \frac{360}{8}$ *Divide 360 by 8, the number of sides in an octagon.*

$= 45^\circ$ *Simplify*



6-1 The Polygon Angle-Sum Theorems

Got It ? Solve With Your Partner

Problem 1 **Investigating Interior Angles of Polygons**

What is the sum of the measures of the interior angles of an octagon?

$n = 8$

$(8 - 2)180^{\circ}$

1080°

6-1 The Polygon Angle-Sum Theorems

Got It ? Solve With Your Partner

Problem 2 **Finding a Polygon Angle Sum**

a. What is the sum of the interior angle measures of a 17-gon?

2700^{o}

b. The sum of the interior angle measures of a polygon is 1980^{o}. How can you find the number of sides in the polygon?

Answers may vary.

Sample: Divide 1980 by 180^{o} and then add 2.

6-1 The Polygon Angle-Sum Theorems

Got It ? Solve With Your Partner

Problem 3 **Using the Polygon Angle-Sum Theorem**

a. What is the sum of the measures of the interior angles of a regular nonagon? Explain.

The sum is 1260^{o} because $(9 - 2)180^{o}$.

b. What is the measure of each interior angle in a regular nonagon?

140^{o}

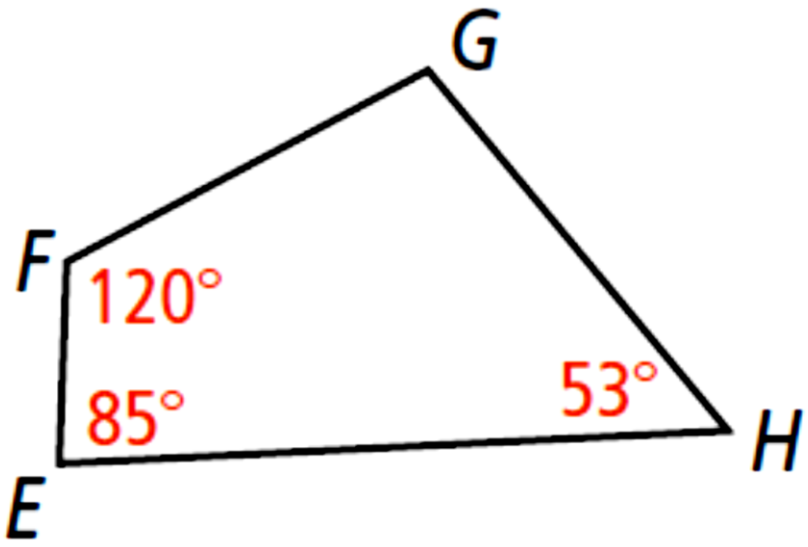
6-1 The Polygon Angle-Sum Theorems

Got It ? Solve With Your Partner

Problem 4 **Using the Polygon Angle-Sum Theorem**

What is $m\angle G$ in quadrilateral *EFGH* ?

102°



6-1 The Polygon Angle-Sum Theorems

Got It ? Solve With Your Partner

Problem 5 **Finding an Exterior Angle Measure**

a. What is the sum of the exterior angles for a regular nonagon?

360^o

b. What is the measure of an exterior angle of a regular nonagon?

40^o

6-1 The Polygon Angle-Sum Theorems

Closure: Communicate Mathematical Ideas (1)(G)

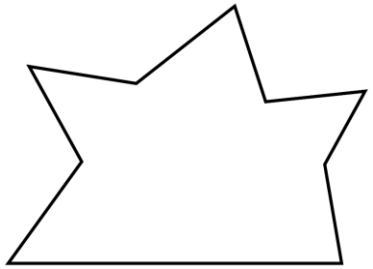
- **For what *n*-gon is the sum of the measures of the interior angles equal to the sum of the measures of the exterior angles? Explain.**

The sum of the exterior angles is always 360^o, so find an *n*-gon such that $(n - 2)180^o = 360^o$. Solving this equation gives $n = 4$. A quadrilateral has the same sum for the measures of the interior angles and the measures of the exterior angles.

6-1 The Polygon Angle-Sum Theorems

Exit Ticket:

1. Name the polygon by the number of its sides. Then tell whether the polygon is regular or irregular, concave or convex.



nonagon; irregular; concave

2. Find the sum of the interior angle measures of a convex 11-gon.

1620°

3. Find the measure of each interior angle of a regular 18-gon. 160°

4. Find the measure of each exterior angle of a regular 15-gon. 24°