5-2 Midsegments of Triangle

Do Now

Lesson Presentation

Exit Ticket

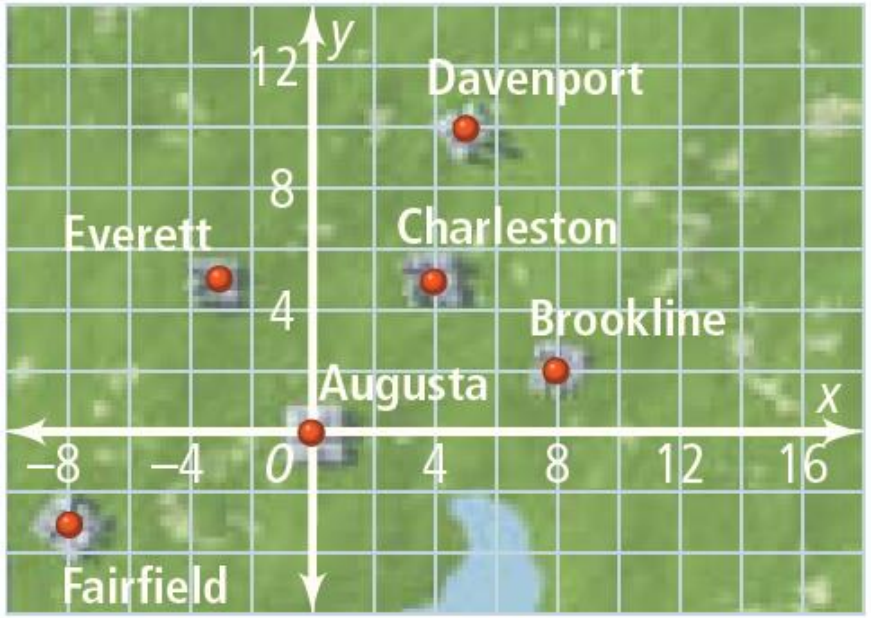
5-2 Midsegments of Triangle

Warm Up #6

1. **Use Representations to Communicate Mathematical Ideas (1)E**

   Find the distance between two cities: Augusta and Brookline.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



8.2

2. **Find the midpoint of $\overline{RS}$ is where the endpoints are *R*(−8, 9) and *S*(0, −7). What are the coordinates of *R*?**

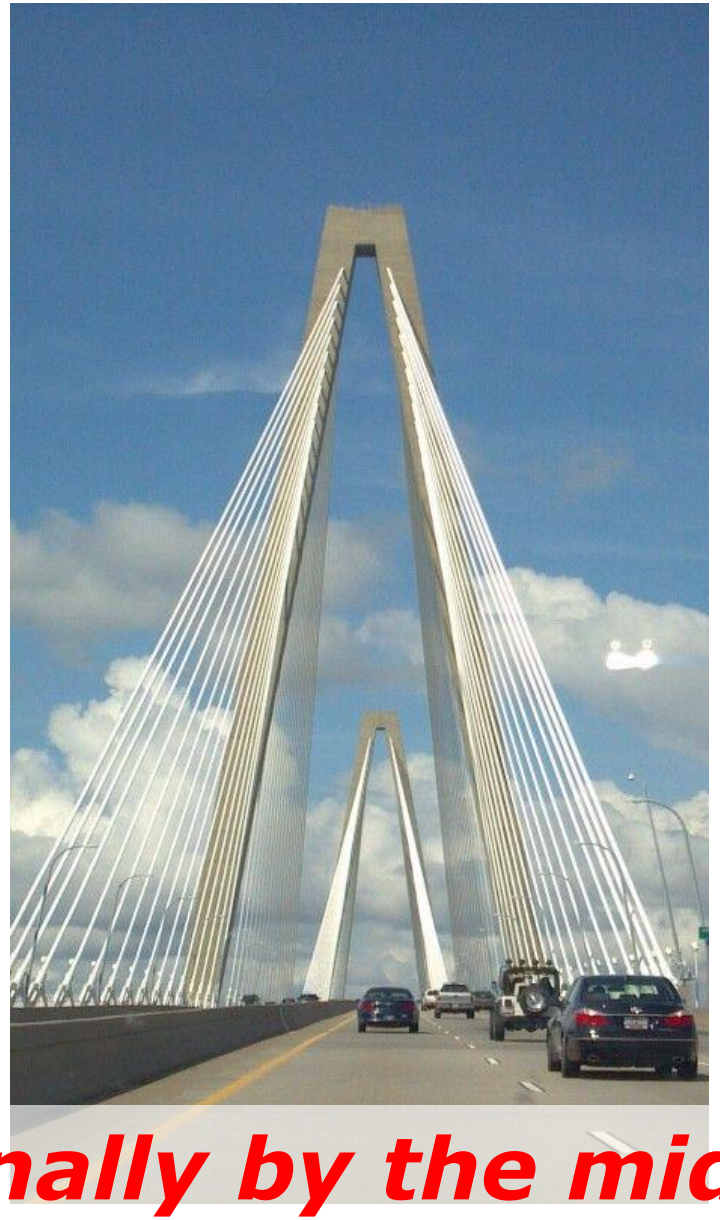
N(−4, 1)

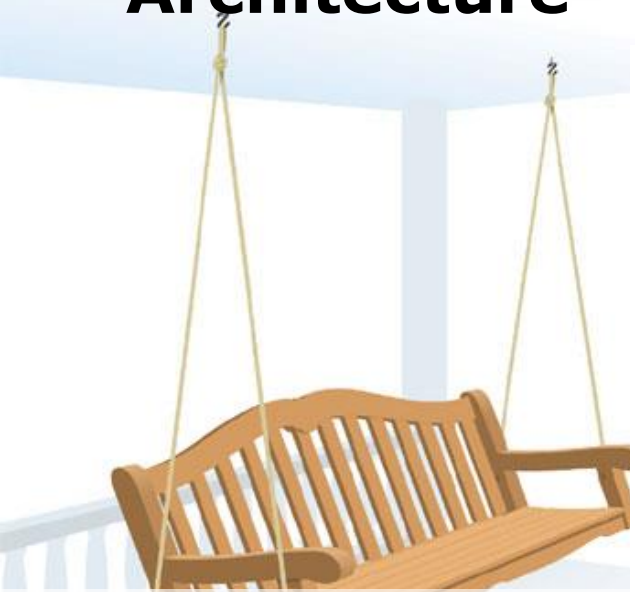
5-2 Midsegments of Triangle

- Today we are going to learn about another part of triangles that also has a connection to similarity.
- We are learning about something called a ***midsegment***.
- Midsegments can be found in many support structures that triangles are used in applications in the world.

5-2 Midsegments of Triangle

- A frame house
- Ladder
- Bridge
- Porch swing
- Ferris wheel
- Architecture





All supported additionally by the midsegment bars.

5-2 Midsegments of Triangle

Connect to Mathematical Ideas (1)(F)

By the end of today's lesson,

SWBAT

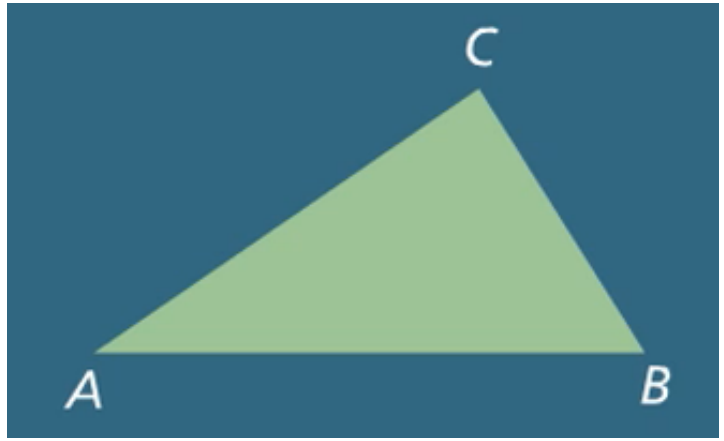
- Solve for side lengths and angle measures in similar triangles with midsegments.

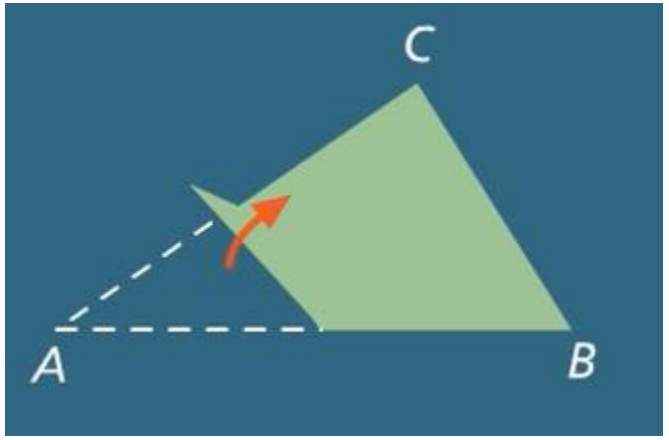
Why it matters in LIFE: Midsegments support many structures in our world like the floors of a house, ladder, or bridge.

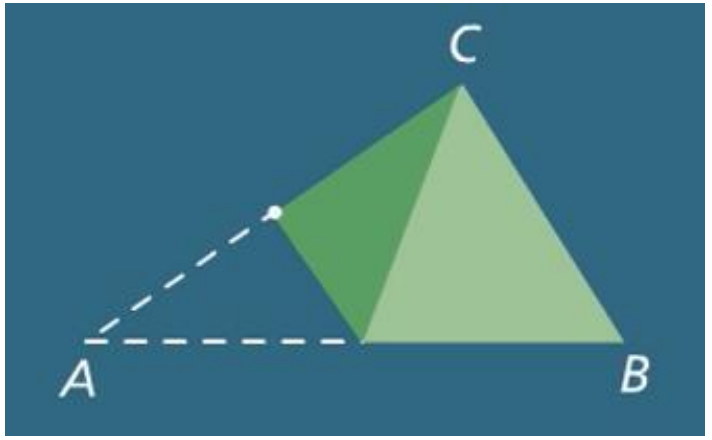
Why it matters in THIS CLASS: Sides and angles are the two fundamentals that make up triangles. Midsegments allow us to solve for both missing sides and angles in triangles.

5-2 Midsegments of Triangle

1. Label your triangle. Label its largest angle *C*, and the other angles *A* and *B*.

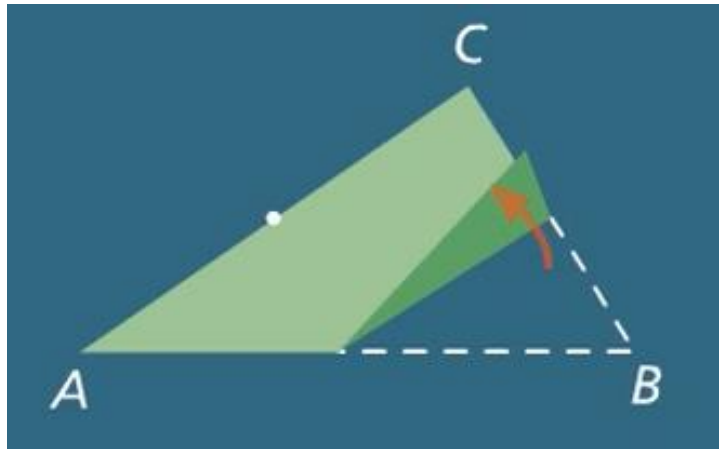


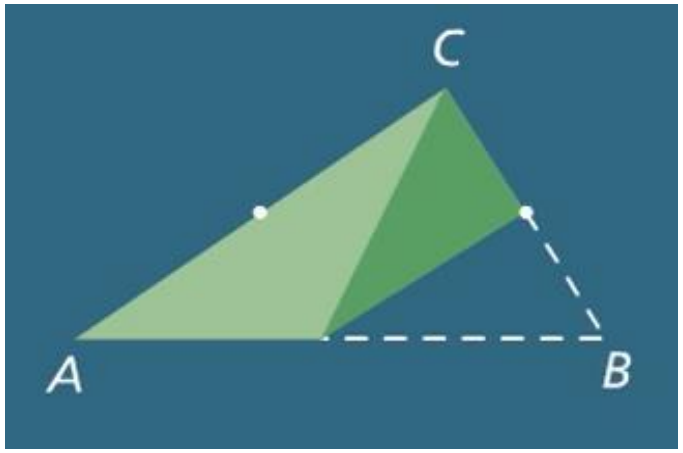


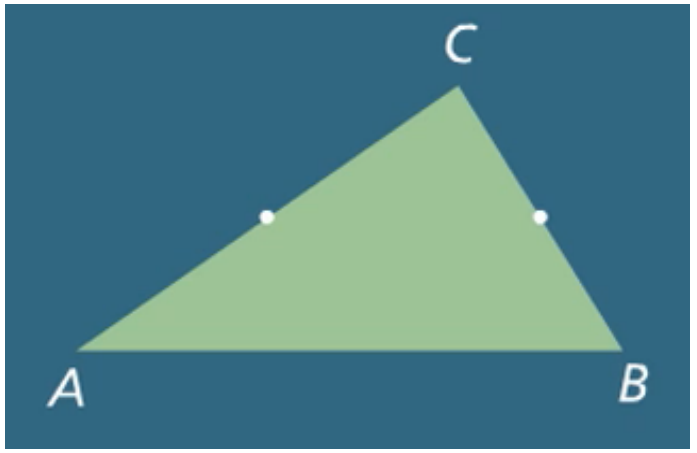


2. Fold *A* onto *C* to find the midpoint of *AC*.

3. Do the same for *BC*.



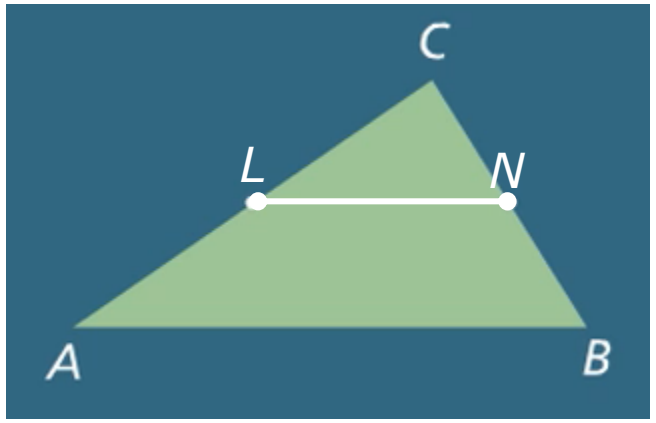


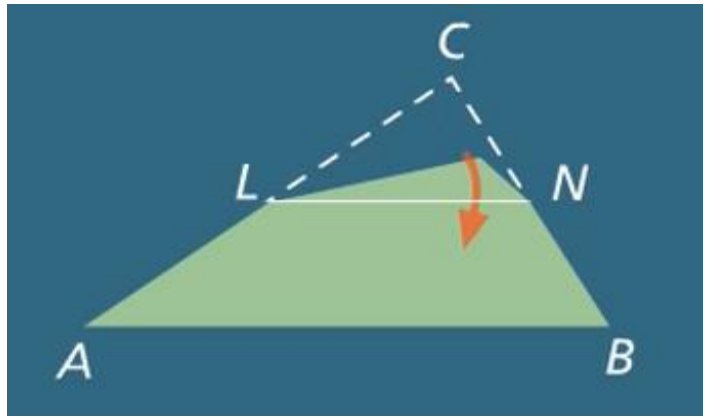


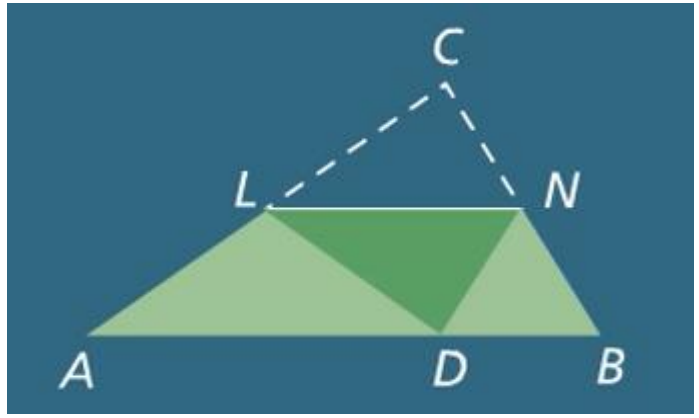
5-2 Midsegments of Triangle

4. Label the midpoints *L* and *N* and draw segment *LN*

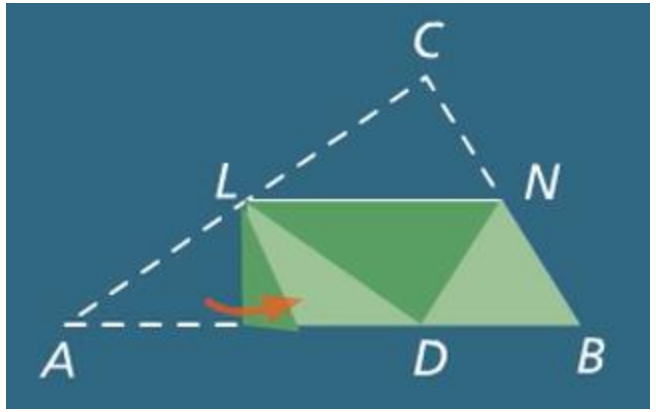
5. Fold the triangle on *LN* and label *D* as shown.

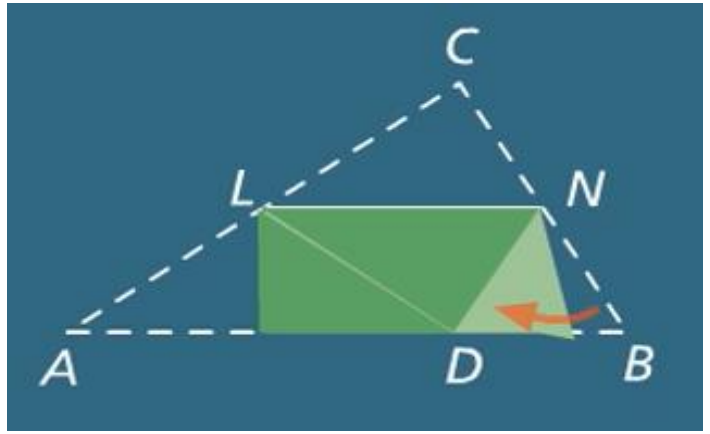


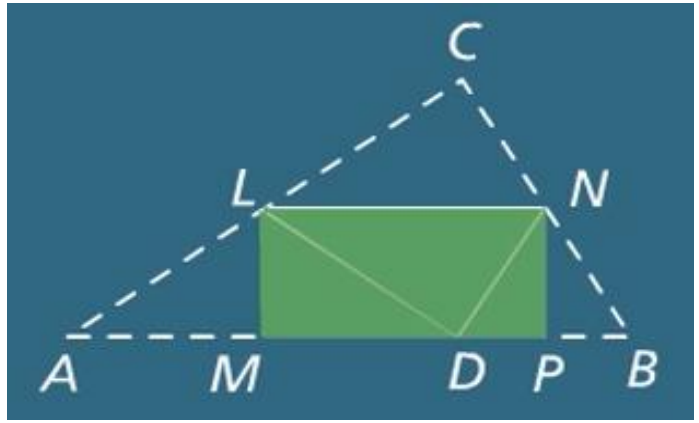




6. Fold *A* to *D* and Fold *B* to *D*. Label the vertices *M* and *P* as shown.

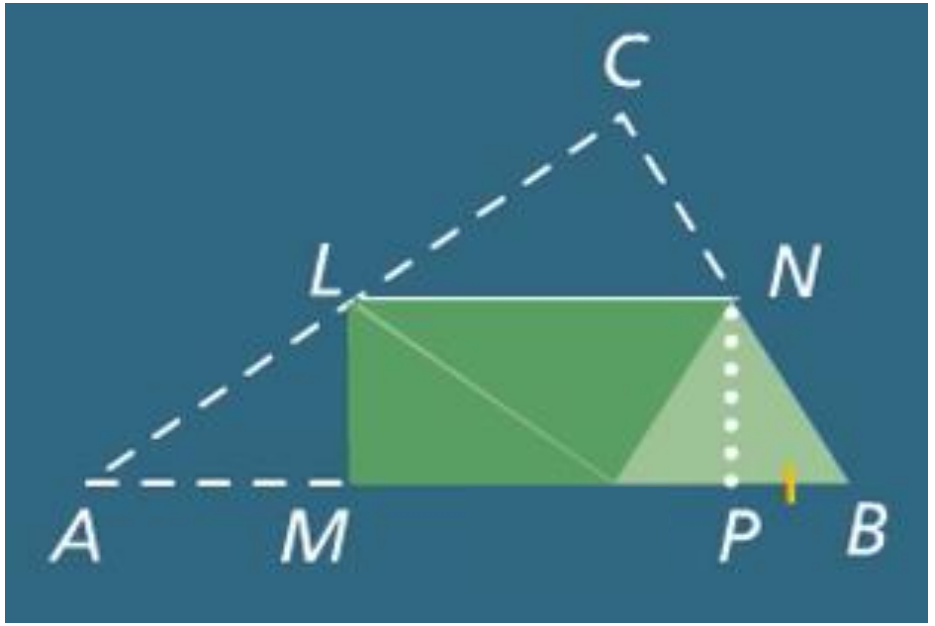






5-2 Midsegments of Triangle

- **What is the relationship between *MP* and *AB*? How do you know?**

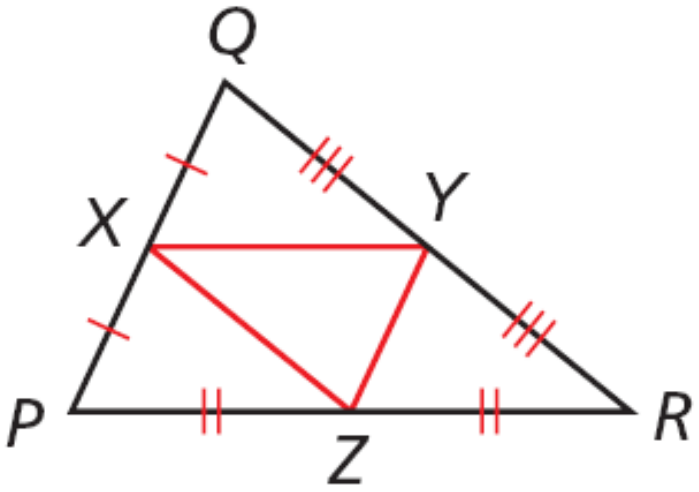


- **What conjecture can you make about the relationship between *LN* and *AB*?**

5-2 Midsegments of Triangle

Vocabulary:

A **midsegment of a triangle** is a segment that joins the midpoints of two sides of the triangle. Every triangle has three midsegments, which form the *midsegment triangle*.



Midsegments: $\overline{XY}$, $\overline{YZ}$, $\overline{ZX}$

Midsegment triangle: $\triangle XYZ$

5-2 Midsegments of Triangle

Connect to Mathematical Ideas (1)(F)

take note

Theorem 5-1 Triangle Midsegment Theorem

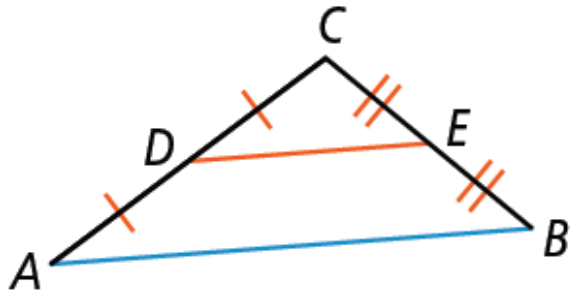
Theorem

If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side and is half as long.

If . . .

D is the midpoint of $\overline{CA}$ and

E is the midpoint of $\overline{CB}$



Then . . .

$\overline{DE} \parallel \overline{AB}$ and

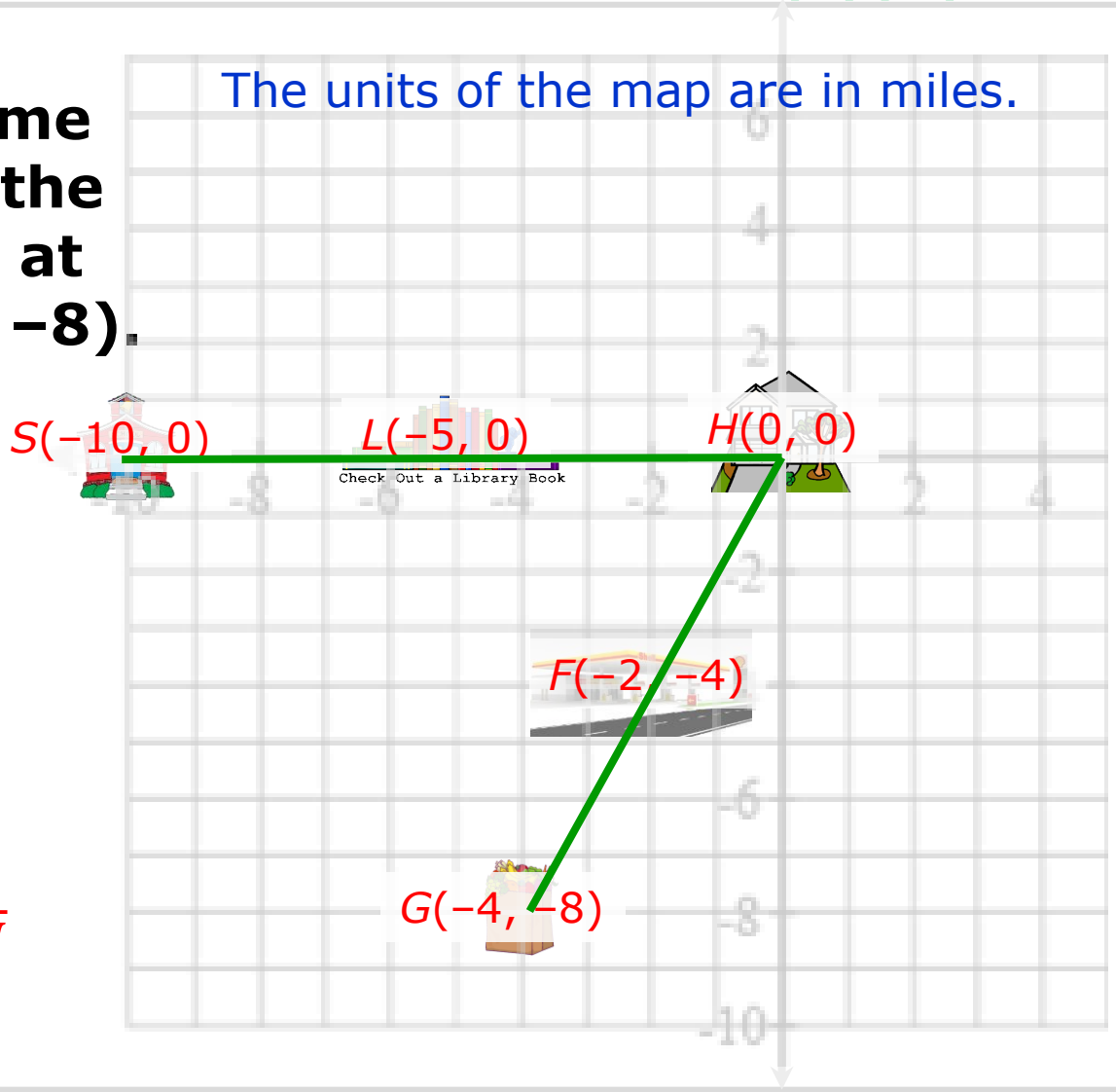
$DE = \frac{1}{2}AB$

For a proof of Theorem 5-1, see Lesson 7-3.

5-2 Midsegments of Triangle

Prior Knowledge: Communicate Mathematical Ideas (1)(G)

The coordinate plan at the right is the map showing the locations of your home at the origin, the school at *S*(–10, 0), the library is at *L*(–5,0), Shell Gas Station at *F*(–2, –4), and the grocery is at *G*(–4, –8).



A. What do you notice about the location of the library?

- Half the distance of $\overline{SH}$
- $L\,(-5, 0)$ is the midpoint of $\overline{SH}$

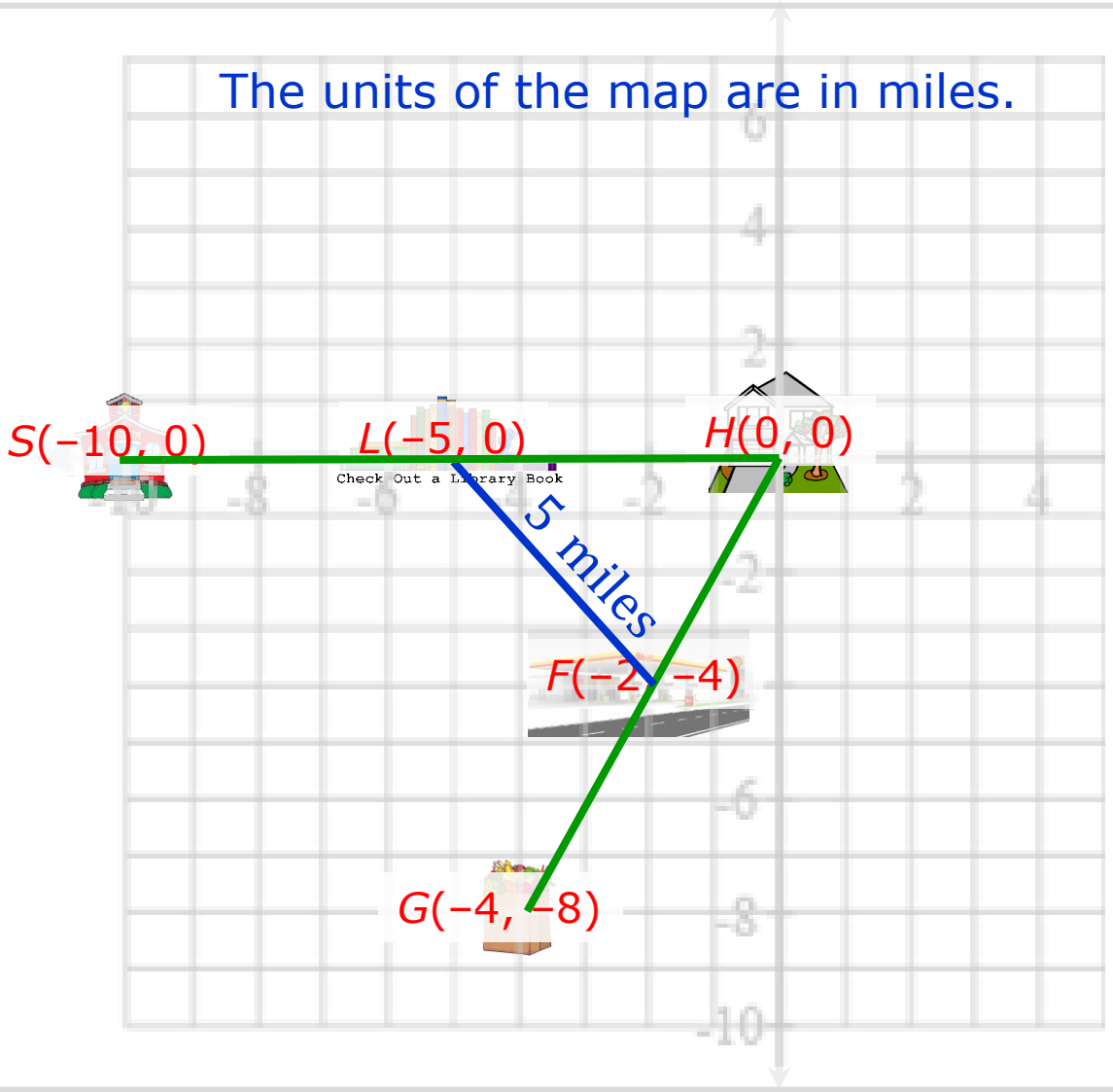
B. What do you notice about the location of Shell Gas Station?

- $F\,(-2, -4)$ is the midpoint of $\overline{GH}$
- Half the distance of $\overline{GH}$

5-2 Midsegments of Triangle

Prior Knowledge: Solve It !!!

Suppose that your parent is picking you up from the school and drop you off at the library, then going to fill up the gas at Shell Gas Station. How far is the library to Shell Gas Station?



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d(\overline{LF}) = \sqrt{(-2 - (-5))^2 + (-4 - 0)^2}$$

$$= \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

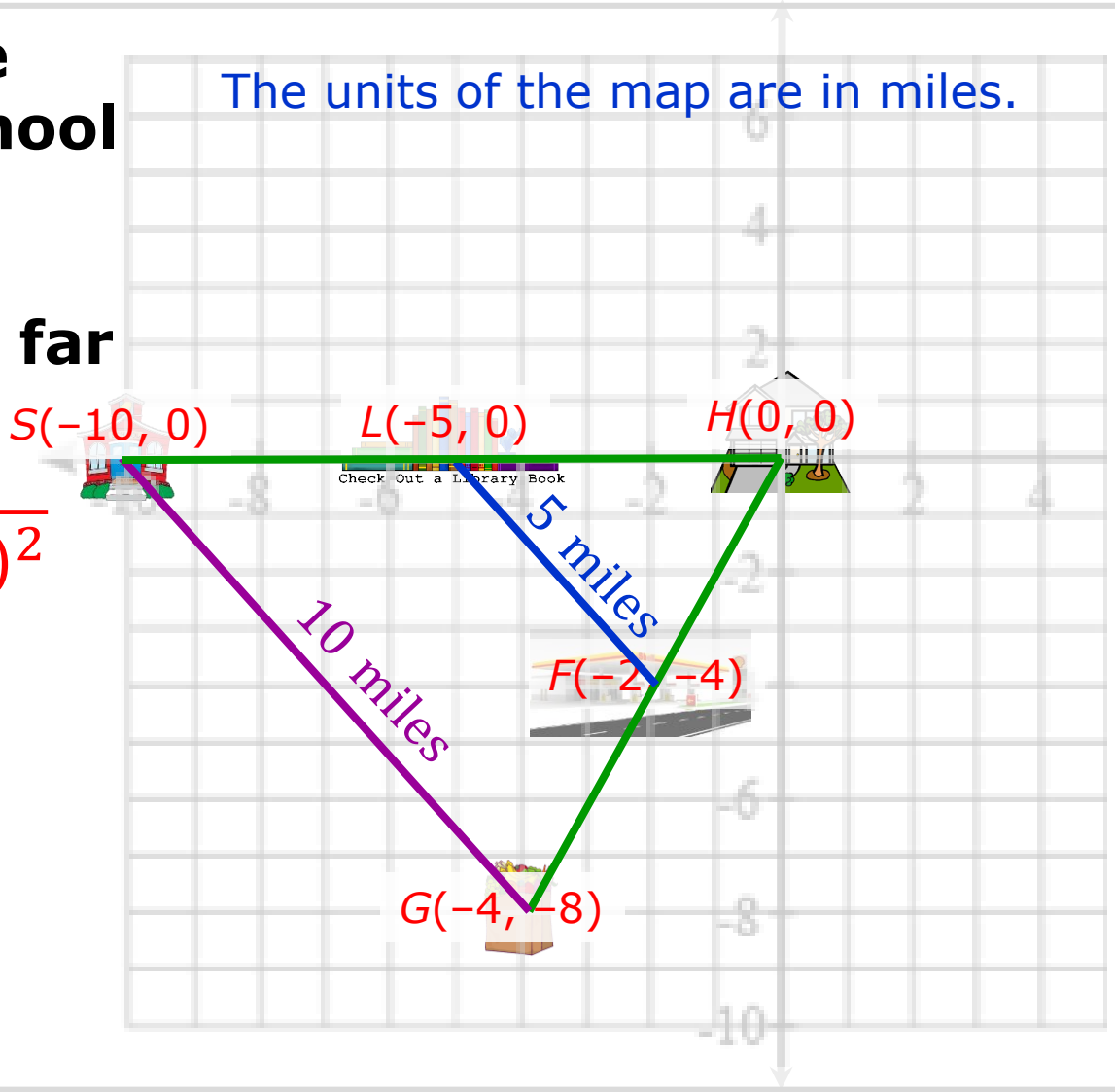
$$= \sqrt{25} \quad \Rightarrow 5 \text{ miles}$$

5-2 Midsegments of Triangle

Prior Knowledge: What if ?

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Your parent decided to pick up some groceries before returning to the school to pick up your older sibling, that is participating in after school math tutoring program, at *S*(–10, 0). How far is the grocery store to the school?



$$d(\overline{SG}) = \sqrt{(-4 - (-10))^2 + (-8 - 0)^2}$$

$$= \sqrt{(6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} \Rightarrow 10 \text{ miles}$$

5-2 Midsegments of Triangle

Connecting and Analyze Mathematical Relationships (1)(F)

1. What is the conjecture for point L?

   L is the midpoint of $\overline{SH}$

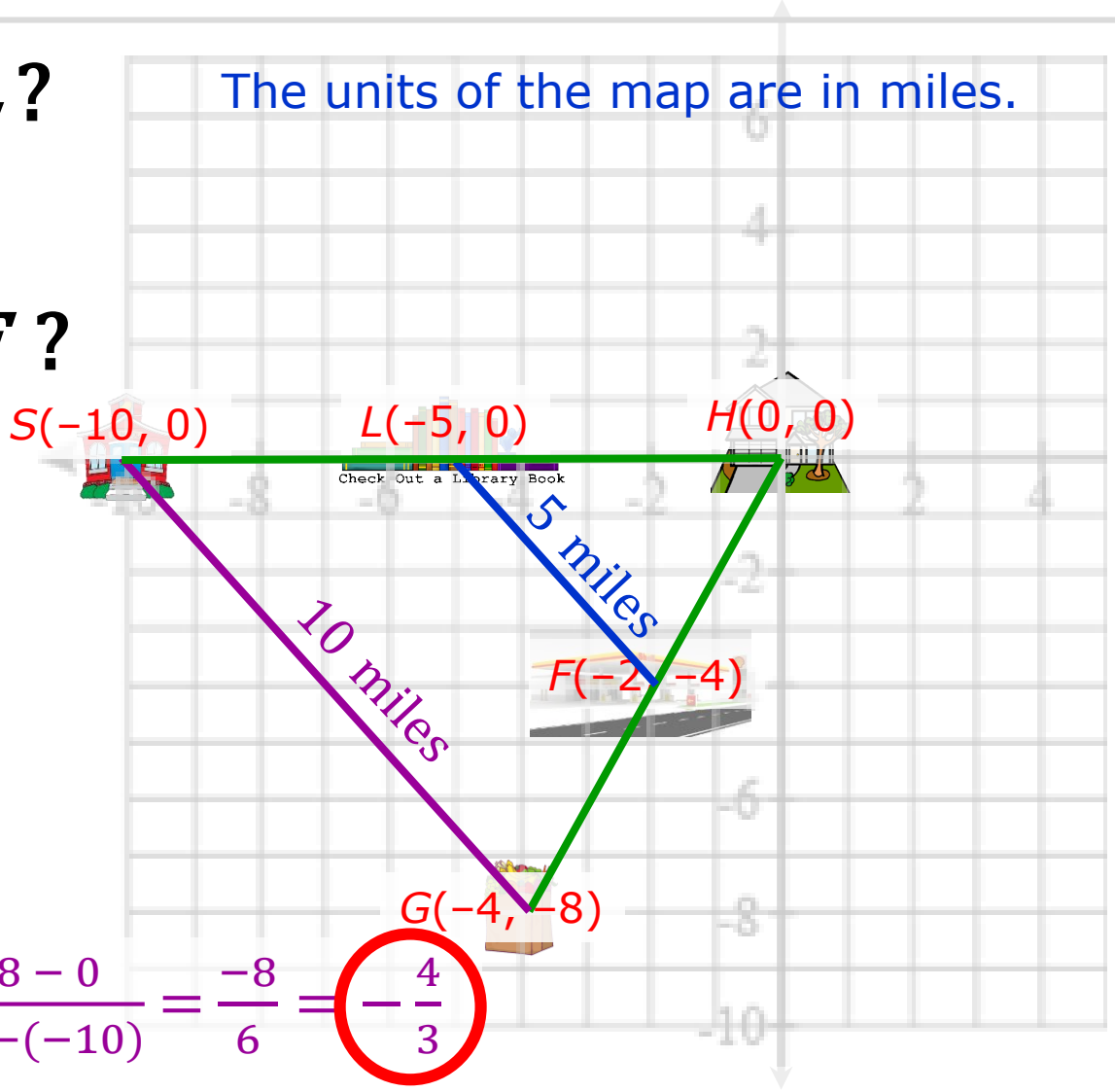
2. What is the conjecture for point F?

   F is the midpoint of $\overline{GH}$

3. Compare segments $\overline{LF}$ and $\overline{SG}$.

   $\overline{LF} = \frac{1}{2}\overline{SG}$

4. Justify if $\overline{LF} \parallel \overline{SG}$. Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$



$m(\overline{LF}) = \frac{-4-0}{-2-(-5)} = \frac{-4}{3} = -\frac{4}{3}$ $m(\overline{SG}) = \frac{-8-0}{-4-(-10)} = \frac{-8}{6} = -\frac{4}{3}$

5-2 Midsegments of Triangle

Example 1: Finding the lengths

Points E, D, and H are the midpoints of the sides of ΔTUV.

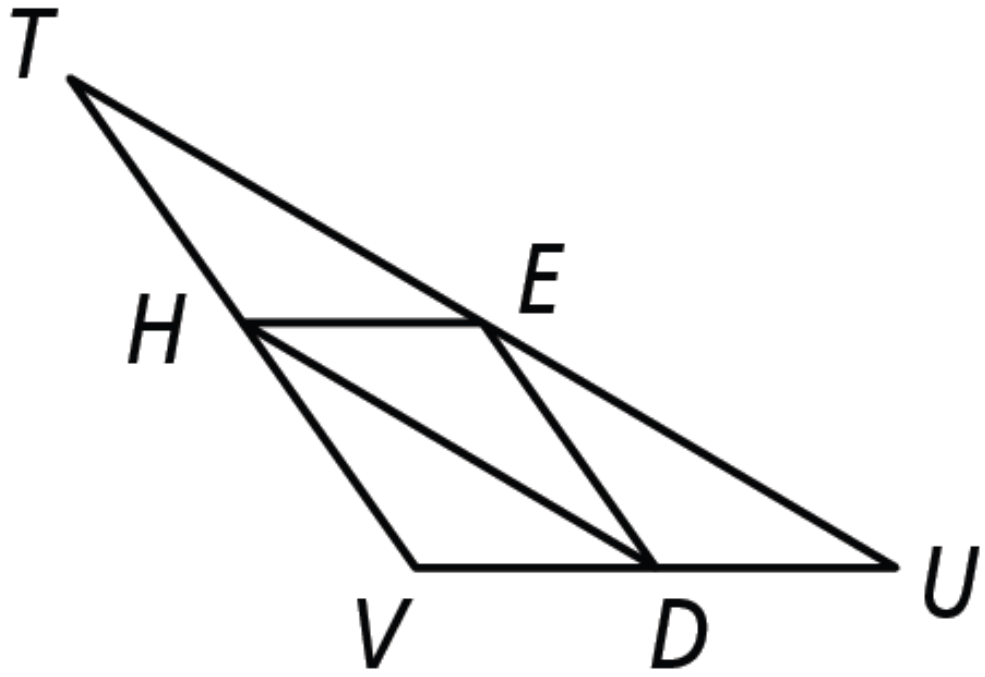
$\overline{UV} = 80, \overline{TV} = 100, \overline{HD} = 80.$

A. Find $\overline{HE}$ 40

B. Find $\overline{ED}$ 50

C. Find $\overline{TU}$ 160

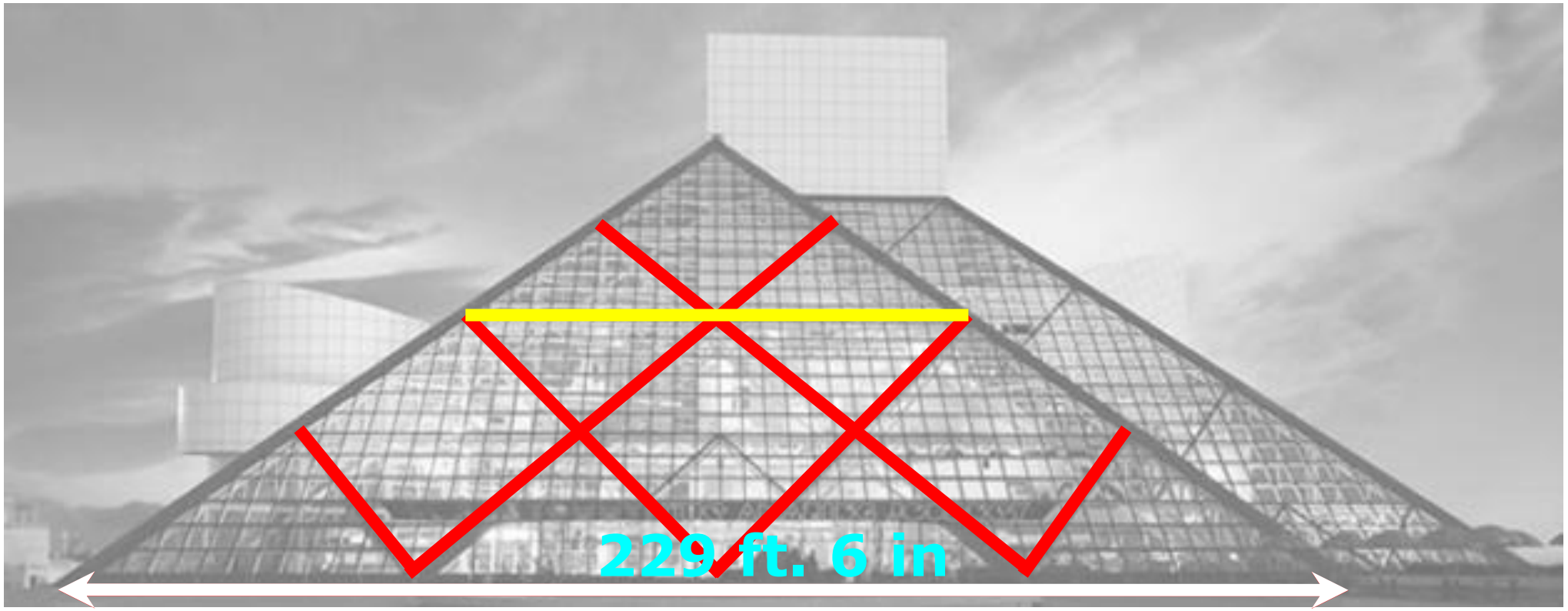
D. Find $\overline{TE}$ 80



5-2 Midsegments of Triangle

Example 2: Explain Mathematical Ideas (1)(G)

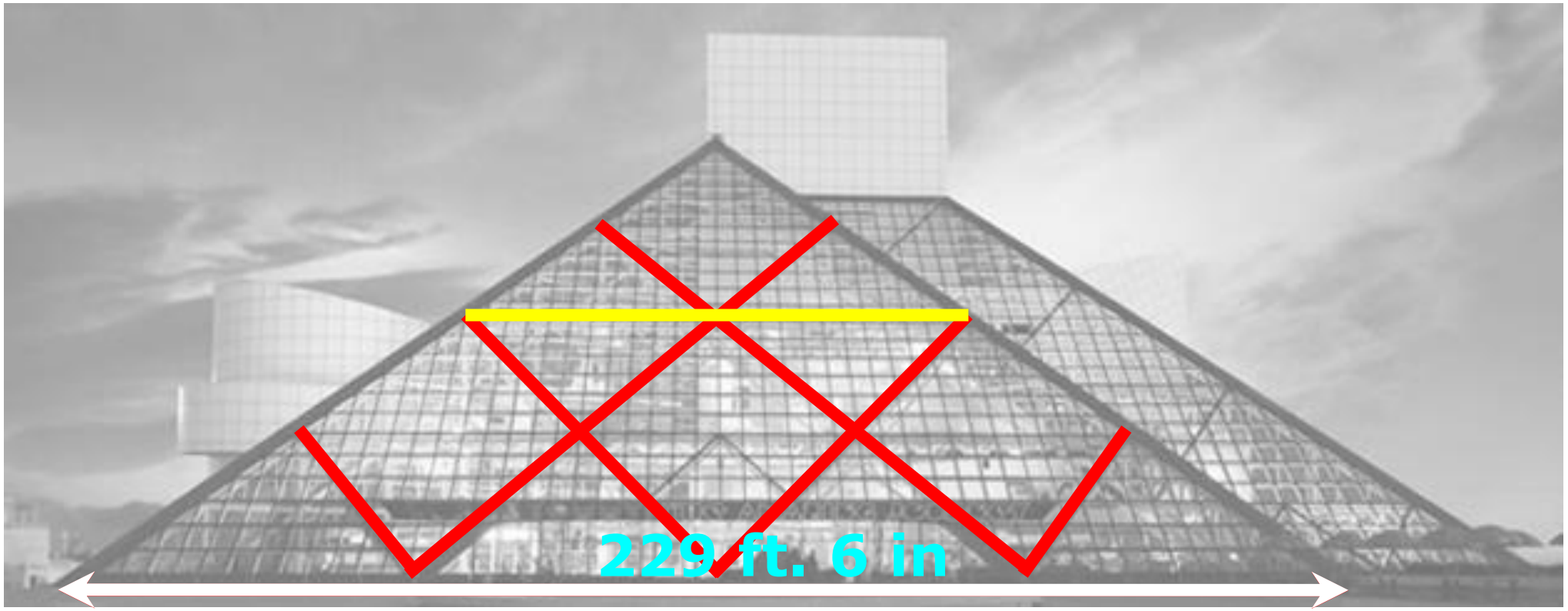
The triangular face of the Rock and Roll Hall of Fame in Cleveland, Ohio, is isosceles. The length of the base is 229 ft. 6 in. Each leg is divided into four congruent parts by the red segments. What is the length of the yellow segment? Explain your reasoning.



5-2 Midsegments of Triangle

Example 2: Explain Mathematical Ideas (1)(G)

It is given that the red segments divide the legs into four congruent parts, the yellow segment is a midsegment of the triangular face of the building, so its length is one haft the length of the base; therefore, the white segment is 114 ft. and 9 in.



5-2 Midsegments of Triangle

Example 3: Use representations to Communicate Mathematical Ideas (1)(E)

You want to paddle your kayak across a lake. To determine how far you must paddle, you pace out a triangle, counting the number of strides, as shown. If your strides average 3.5 ft., what distance must you paddle across the lake?

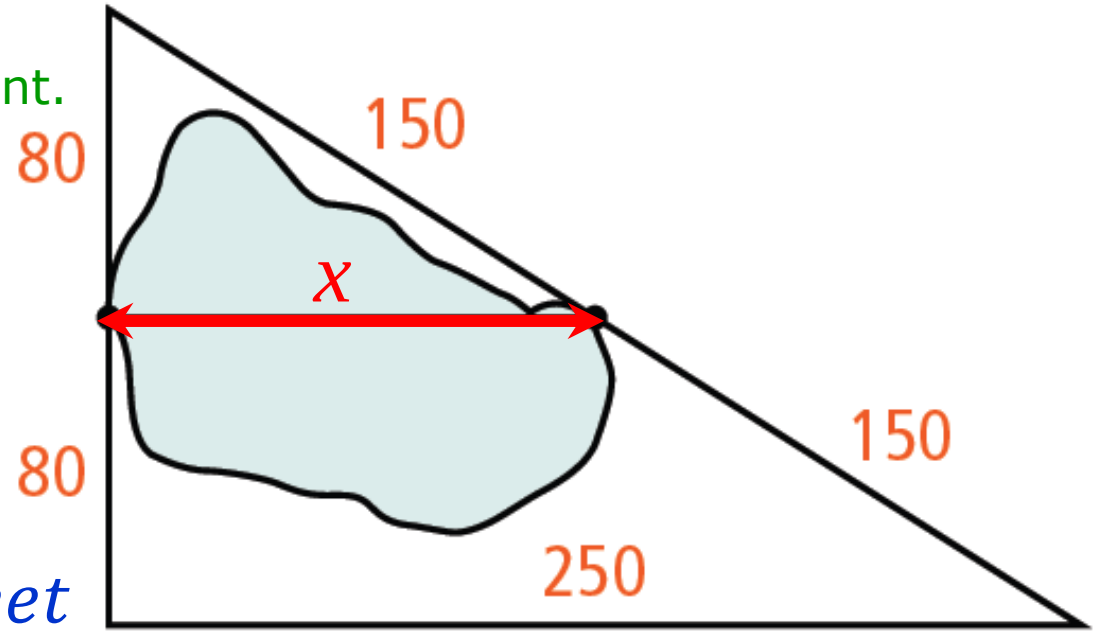
Plan: multiply the strides average by the midsegment.

distance across the lake = $3.5x$

$$x = \frac{1}{2}(250) \qquad d = 3.5(125)$$

$$= 125 \qquad = 437.5$$

the distance across the lake is 437.5 *feet*



5-2 Midsegments of Triangle

Example 4: Finding the value of x.

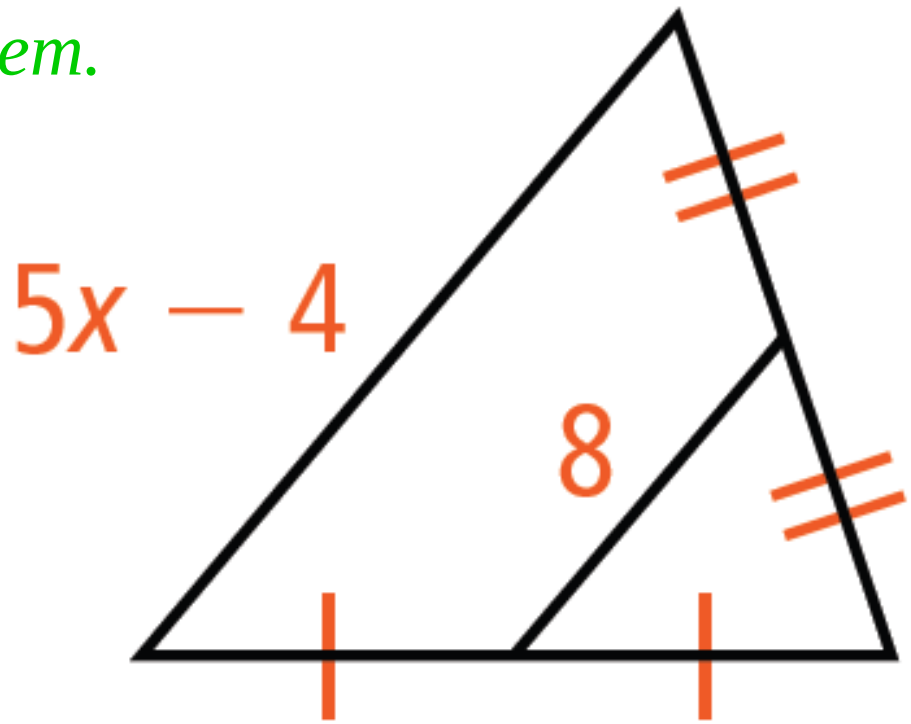
$$8 = \frac{1}{2}(5x - 4)$$ *Δ Midsegment Theorem.*

$$8 = \frac{5}{2}x - 2$$ *Distributive Property*

$$10 = \frac{5}{2}x$$ *Combined Like Terms*

$$20 = 5x$$ *Multiply both sides by 2*

$$4 = x$$ *Divide both sides by 5*



5-2 Midsegments of Triangle

Example 5:

If *EC* = 3*x* – 2 and *AD* = 2*x* + 8. Find *EC*.

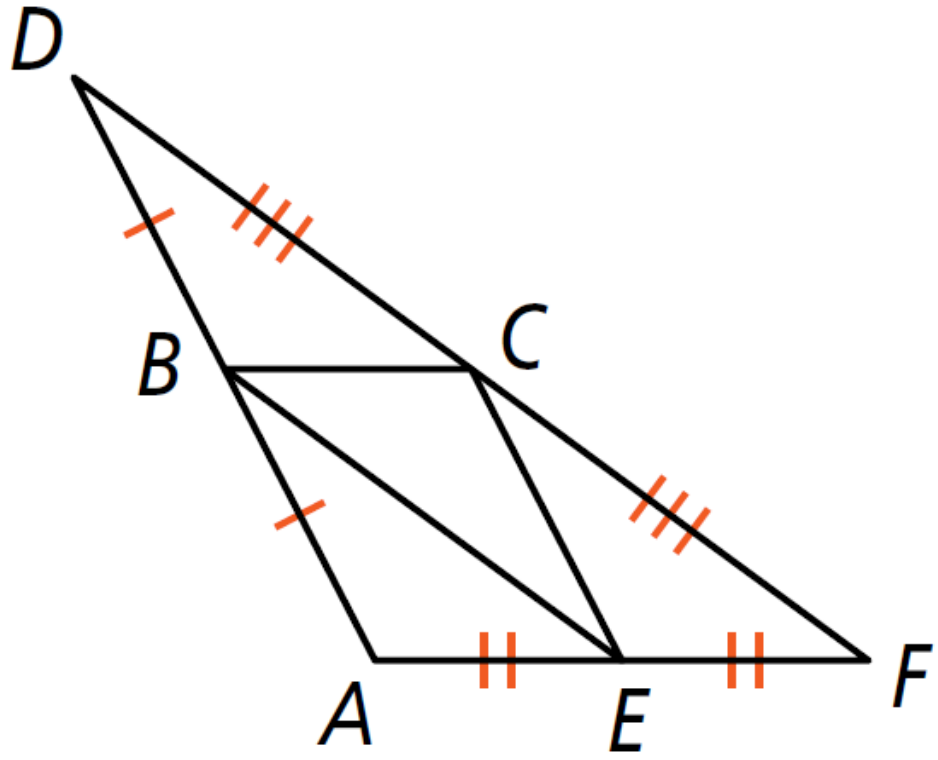
$$EC = \frac{1}{2}AD$$ *Δ Midsegment Theorem*

$$3x - 2 = \frac{1}{2}(2x + 8)$$ *Substitution*

$$3x - 2 = x + 4$$ *Distributive Property*

$$2x = 6$$ *Combined Liked Terms*

$$x = 3$$ *Divide both sides by 2.*



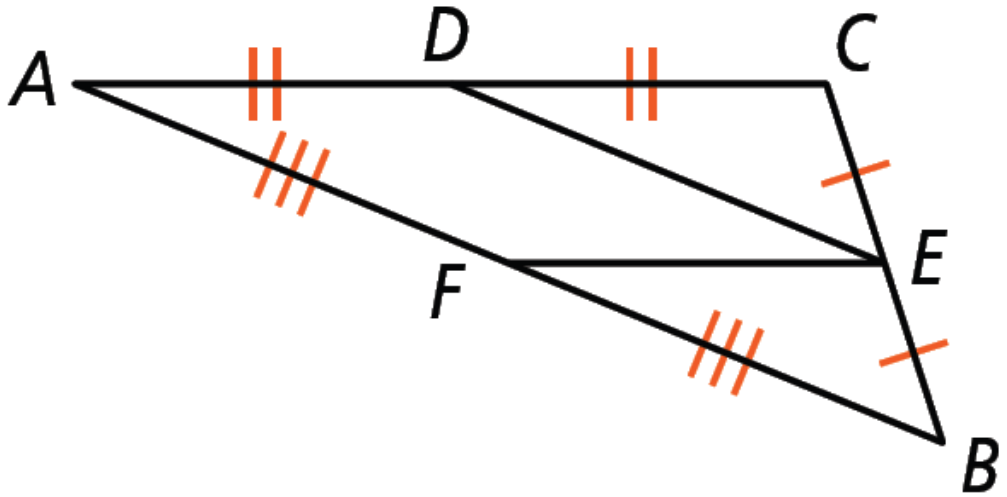
$$EC = 3(3) - 2 \Leftrightarrow 7$$

5-2 Midsegments of Triangle

Got It ? Solve With Your Partner

Problem 1 **Finding Lengths**

In the figure at the below, $AD = 6$ and $DE = 7.5$. What are the lengths of $\overline{DC}$, $\overline{AC}$, $\overline{EF}$, and $\overline{AB}$?



$\overline{DC} = 6$

$\overline{AC} = 12$

$\overline{EF} = 6$

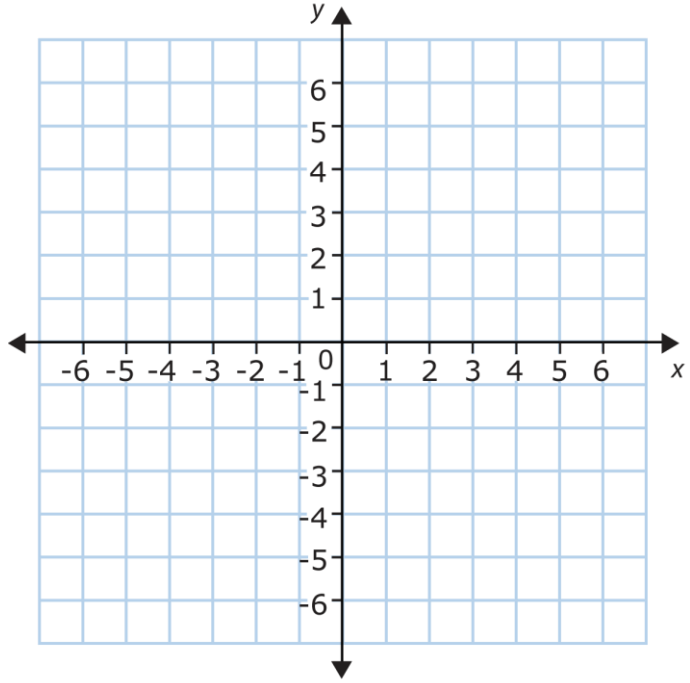
$\overline{AB} = 15$

5-2 Midsegments of Triangle

Got It ? Solve With Your Partner

Problem 2 **Verifying the Triangle Midsegment Theorem**

Verify the Triangle Midsegment Theorem for ΔFGH with vertices $F(-6, 4)$, $G(4,8)$, and $H(2, -2)$. Given that J and K are the midpoints of $\overline{FG}$ and $\overline{FH}$, respectively, show that $\overline{JK} \parallel \overline{GH}$



The midpoint of $\overline{FG}$ is $J\left(\frac{-6+4}{2}, \frac{4+8}{2}\right) = J(-1, 6)$

The midpoint of $\overline{FH}$ is $K\left(\frac{-6+2}{2}, \frac{4+(-2)}{2}\right) = K(-2, 1)$

The slope of $\overline{JK} = \frac{1-6}{-2-(-1)} = \frac{-5}{-1} = 5$

The slope of $\overline{GH} = \frac{-2-8}{2-4} = \frac{-10}{-2} = 5$

So $\overline{JK} \parallel \overline{GH}$

$\overline{JK} = \sqrt{(-2-(-1))^2 + (1-6)^2} = \sqrt{1+25} = \sqrt{26}$

$\overline{GH} = \sqrt{(2-4)^2 + (-2-8)^2} = \sqrt{4+100} = \sqrt{104} = 2\sqrt{26}$

So $\overline{JK} = \frac{1}{2}\overline{GH}$

5-2 Midsegments of Triangle

Lesson Check How did you Do?

1. 4
2. Each of the four small triangles has a side congruent to $\overline{JM}$, a side congruent to $\overline{NM}$, and a side congruent to $\overline{JN}$. Therefore, they are congruent by SSS.
3. The surveyor needs to measure $\overline{NM}$ to find $\overline{PQ}$. $\overline{PQ}$ is twice $\overline{NM}$.
4. A midsegment is a segment whose endpoints are the midpoints of two sides of a triangle.
5. The segments are parallel.
6. No; Susan is assuming that L is the midpoint of $\overline{OT}$, which is not given.

5-2 Midsegments of Triangle

Closure: Communicate Mathematical Ideas (1)(G)

- **In the coordinate plane, how are the slopes of a midsegment and the third side of the triangle related? Explain.**

  The slopes are the same because the two segments are parallel.

- **How is the length of a midsegment of a triangle related to the length of the third side?**

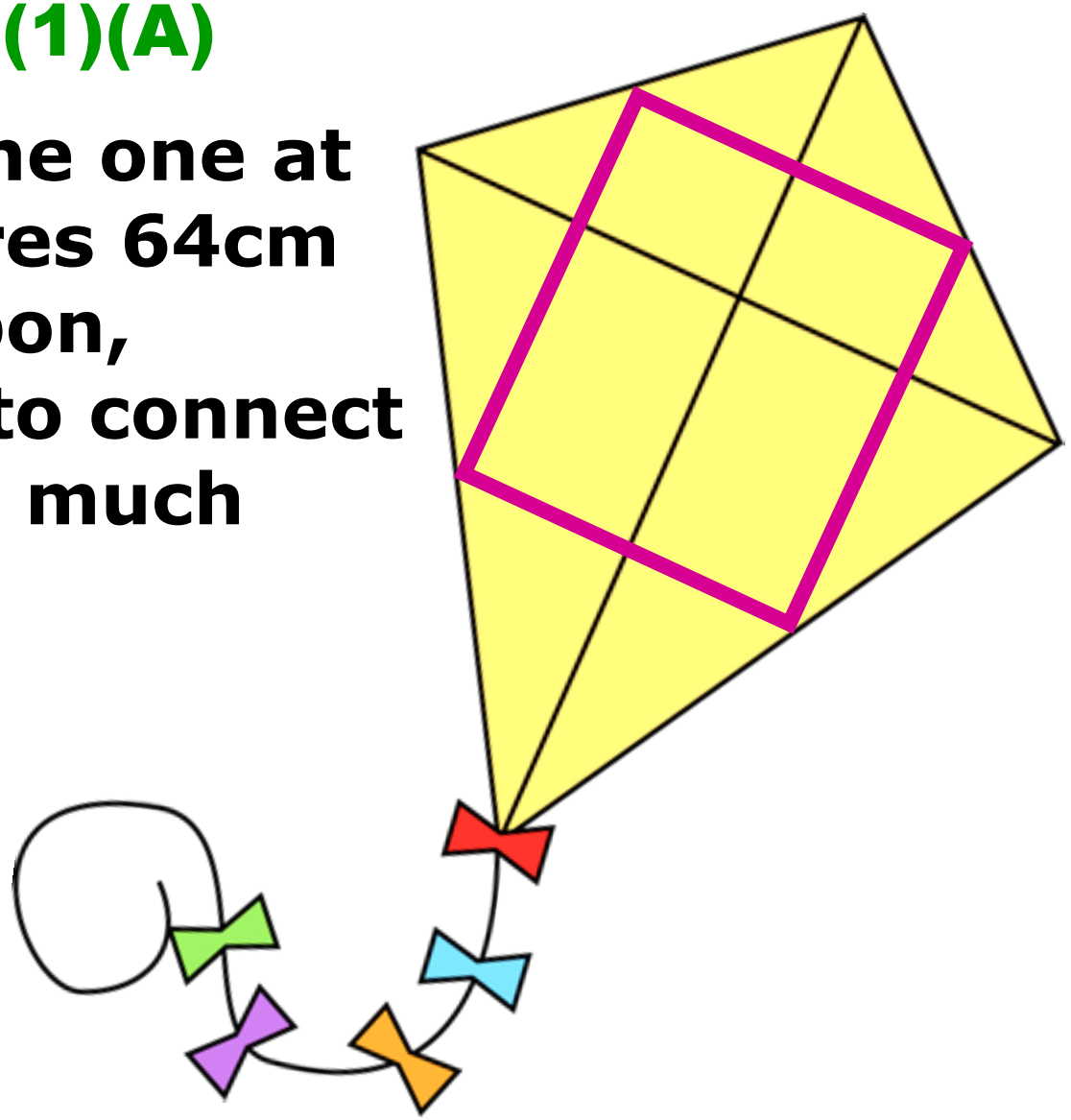
  The length of the midsegment is half the length of the third side.

- **Describe the properties of midsegments of a triangle.**
  - The endpoints of the segments are at the midpoints of the sides of the triangle.
  - The slopes of the midsegment and the third side of the triangle must be the same.

5-2 Midsegments of Triangle

Exit Ticket: Apply Mathematics (1)(A)

You design a kite to look like the one at the right. Its diagonals measures 64cm and 90cm. You plan to use ribbon, represent in purple rectangle, to connect the midpoints of its sides. How much ribbon do you need?



A. 77 cm

C. 154 cm

B. 122 cm

D. 308 cm